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Gravity mediated entanglement between light beams as a table-top test of quantum gravity

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Abstract

A unified framework for quantum gravity has been highly sought after by a large community within theoretical physics for about a century. Yet, to date, there is no experimental evidence for any non-classical features of gravity. While experimental proposals would usually require high-energy setups that are often considered far from feasible, recent table-top test protocols are believed to be within technological reach in the not-too-distant future. The generation of entanglement between two massive objects interacting gravitationally has been proposed as an indirect witness for non-classical features of gravity as the mediator of the entanglement. This highly vibrant research field is driven by a lively discussion on both the possible implications of gravitationally mediated entanglement as well as feasibility considerations for two massive objects. As an alternative platform, this dissertation examines the use of two counter-propagating light beams interacting gravitationally to eliminate some of the problems the massive protocol suffers from. Even though further work is still required, a photonic realisation of gravity mediated entanglement brings with it several advantageous features unmatched by its massive counterpart.

Acknowledgments

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Contents

Chapter 1

Introduction

At the beginning of the 20^{th} century, physics went through a revolutionary paradigm shift with the birth of its two main pillars, General Relativity and Quantum Theory. While the former is the current best theory describing gravity, the latter provided a means to establish the Standard Model of particle physics, which is the current Quantum Field Theory to describe the three other known interactions (electromagnetism, weak and strong force) as well as the elementary particles.

Experiments have agreed with the predictions of the two theories in their respective domains to very high accuracy, but it is still an open question how exactly to incorporate the two theories into a common framework, what is vaguely labelled as quantum gravity. The hope is to give a more comprehensive account of gravity valid at very high energies or small distances [1]. Often general relativity is described as the theory of the very big and quantum theory as the theory of the very small scales. Yet, this is not necessarily true, and there are situations where understanding both frameworks is paramount. This would include better insights into early universe cosmology and black holes as well as the unification of all interactions [2]. This enterprise of formulating such a theory resulted in various approaches, for example string theory [3, 4], loop quantum gravity [5], causal set theory [6] and many others. This would, at least in principle, allow for a description of what happens at scales as large as the diameter of the observable universe with $8.8 \cdot 10^{26}$m down to the Planck length $l_p = \sqrt{G\hbar/c^3} \approx 1.6 \cdot 10^{-35}$m [7]. There are many different approaches based on quantising gravity or gravitising quantum theory, often with their internal inconsistencies up until now.

However, all these approaches have been unguided by experimental progress, but suggest very specific predictions. In the intermediate regime, gravity is mostly insignificant, for example in particle physics experiments. Even then, a Newtonian description is completely sufficient. With the Planck scales being extremely far away from current experimental probes, the tale often goes that there is no means of carrying out any experiment at hand that would rule out or favor any of the proposed quantum gravity approaches. It is hoped that this will change and direct theoretical research. The natural arena of experiments would have been related to black holes or extreme high-energy accelerator experiments.

Most conventional proposed experimental tests [8, 9] of quantum gravity include the evaporation of black holes, quantum gravity corrections to the CMB or perhaps tests on the discreteness of space. Regardless, the key for obtaining empirical hints lies perhaps in table-top experiments at low energies. Noteworthy is also that the Planck mass, as opposed to the Planck energy, is with a few micrograms within experimental reach to probe quantum gravity effects [10]. While having switched from high-energy physics to low-energy physics in table-top tests allows for simpler experimental implementation, one is now only considering weak quantum gravity. Many fundamental theories of quantum gravity may share the same phenomenology. The advantage of this approach is that table-top tests do not rely on testing a specific model of quantum gravity, but they are often model-agnostic [11].

A majority of the scientific community seeking for a quantum theory of gravity [12] declares finding a quantum description of gravity as just a technical problem. On the other hand, it is not entirely clear whether gravity is really a quantum entity in nature, or at least it has not yet been shown empirically. According to some researchers in the high-energy physics community [7], the claim that gravity does not necessarily need to be quantised is implausible given what the history of physics has shown us over the last century, with quantum theory being the most fundamental. This argument is based on nature being one, and so we must also unify gravity with quantum theory facilitated by the universality of the two theories. However, proposals that model gravity as a fundamentally classical entity exist [7]. For example, one could consider Einstein's equations relating the Einstein tensor $G_{\mu\nu}$ to the expectation value of the quantised stress-energy tensor operator $\hat{T}_{\mu\nu}$:

$$G_{\mu\nu} = \frac{8\pi G}{c^4}\langle \hat{T}_{\mu\nu} \rangle \tag{1.1}$$

where G is the gravitational constant and c is the speed of light.

The history of table-top test proposals claiming to probe the quantum nature of gravity is rich [11, 13]. Most notably, the experimental proposals by Bose, Marletto, Vedral et al [14, 15] introduced the idea of coupling two massive spatially separated superposition states (i.e. Schrödinger cat states) gravitationally. The mediated entanglement is then understood as evidence for the quantum nature of gravity. This experimental proposal has sparked much interest in the community. To enrich the discussion, the proposal introduced in this dissertation marries the existing protocol with the idea that light beams may also be used as sources of gravity mediated entanglement [16, 17]. While the dissent on the implications of the original protocol on the conclusions to be drawn on the nature of gravity is still present, the investigation of the gravitational coupling of two light beams by witnessing entanglement would be an experimental achievement by itself.

This dissertation is organised as follows: In section 2, the original protocol for gravity mediated entanglement in the case for two massive objects is reviewed. A discussion is given on the experiment's limitations and criticism as well as current suggestions of extensions or variations. In Section 3, we present an experimental implementation using counter-propagating light beams in Mach-Zehnder interferometers. While the gravitational field induced by light pulses has been studied in the literature, we extend and implement the framework using linearised gravity for the use in witnessing gravity mediated entanglement. After obtaining expressions for the phases being imprinted by gravity, we discuss the validity of our calculations. Finally, we conclude by giving an outlook for further research.

Chapter 2

Reviewing the experimental proposal on gravity mediated entanglement

When in 2017 almost simultaneously two papers were published independently by different authors, quantum gravity as a field seemed to make a leap to become subject to experimental investigation in the near future. The original protocols [14, 15] suggest to settle the question of whether gravity is a quantum entity or not experimentally in a table-top experiment witnessed by gravity mediated entanglement (GME). To understand their relevance and especially our contribution to the subject by replacing massive objects with light beams in Section 3, we hereby outline the original proposal below.

2.1 Outline of the original GME protocol

Let us consider the originally proposed experimental setup with two massive test objects of mass m each put into a Mach-Zehnder interferometer, or Stern-Gerlach interferometer alternatively[1], as illustrated in Fig. 2.1. Initially at time t_i, the two masses are spatially separated and prepared in well-defined motional states. During the *splitting* stage ($t_i < t \leq 0$) each mass is put into a spatial superposition that is denoted schematically $|+\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$, where $|L/R\rangle$ are the left/right spatial eigenstates. After the co-propagation of the two masses during *free-fall* ($0 < t \leq \tau$), the masses are finally brought again back to their initial spatial eigenstates during *refocusing* ($\tau < t \leq t_f$).

The different quantum mechanical phases are induced due to the gravitational interaction, which is a function of distance. We assume that the time required for splitting and refocusing is negligible compared to the free-fall interaction time τ for simplicity[2].

[1] more generally any type of two adjacent matter-wave interferometers

[2] The phase evolution due to splitting and refocusing can also be calculated explicitly to obtain a

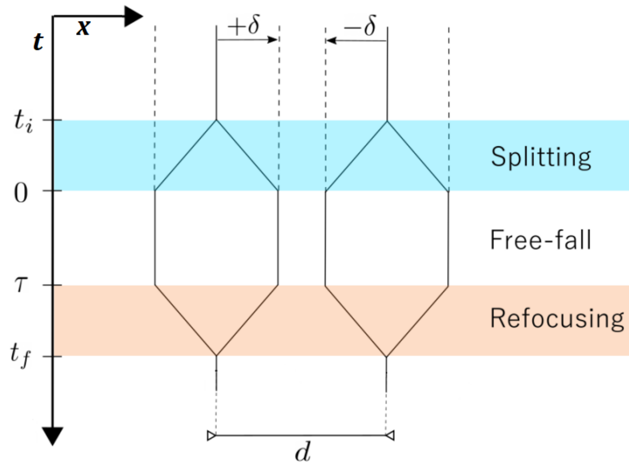


Figure 2.1: Schematic illustrating the GME protocol setup for two massive co-propagating objects. The centres of the two matter-wave interferometers are separated by a distance d and the symmetric splitting causes each of the spatial branches to deviate by a distance δ from the centre of each interferometer. Figure adapted from [18].

To ascertain the position eigenstates during free-fall, the gravitational deflection ought to be negligible as well. As shown for the original protocol, the position eigenstate approximation [18] is indeed valid for common experimental parameter regimes.

In the original protocols, the gravitational interaction is assumed to be Newtonian with interaction energy $-G\frac{m^2}{d_{ab}}$, where $G = 6.67 \cdot 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ is the gravitational constant and d_{ab} is the distance of the two objects at positions a, b. The quantum mechanical phase due to gravity ϕ_{ab} for two positions a, b after interaction time τ is hence given by

$$\phi_{ab} = G\frac{m^2}{\hbar d_{ab}}\tau, \tag{2.1}$$

where $\hbar = 1.05 \cdot 10^{-34}\mathrm{m}^2\mathrm{kgs}^{-1}$ is Planck's constant. The state containing the relevant spatial degrees of freedom will evolve from the initial state

$$|\psi(0)\rangle = |+\rangle_1 \otimes |+\rangle_2 = \frac{1}{2}(|L\rangle_1 + |R\rangle_1) \otimes (|L\rangle_2 + |R\rangle_2) \tag{2.2}$$

at time $t = 0$ to

$$|\psi(\tau)\rangle = \frac{1}{2}\left(|LL\rangle + e^{i\Delta\phi_{LR}}|LR\rangle + e^{i\Delta\phi_{RL}}|RL\rangle + |RR\rangle\right) \tag{2.3}$$

after interaction time τ. Any global phases have been ignored and we denote $\Delta\phi_{ab} = \phi_{ab} - \phi$, where $\phi = G\frac{m^2}{\hbar d}\tau$ with d being the distance between the centres of the two

more accurate description. Alternatively, the experimental setup can be amended by removing the free-fall stage altogether and entangling the two masses only through the two other stages of the protocol.

matter-wave interferometers. Note that the assumption of localised Gaussian wavepackets that Bose et al [14] invoked ensures the orthogonality of the two spatial eigenstates, i.e. $\langle L|R\rangle = 0$.
Consider the definition of entanglement that we we will use for pure states:

Definition 1 *Let $|\psi_{12}\rangle$ be any pure bipartite state in the Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. If it can be written as a product of states belonging to each subsystem's Hilbert space with $|\psi_a\rangle \in \mathcal{H}_a\,(a \in \{1,2\})$ such that*

$$|\psi_{12}\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \tag{2.4}$$

then it is called separable. Otherwise it is called entangled [19].

Definition 2 *The maximally entangled state for a d-dimensional Hilbert space $\mathcal{H}$ with basis states denoted by $|i\rangle$ is [20]*

$$|\psi\rangle_{maxent} = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i\rangle \otimes |i\rangle. \tag{2.5}$$

Similarly, for density operators, we have the following definitions [21]:

Definition 3 *Let ρ_{12} be any bipartite density operator in the Hilbert space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$. If it can be written in the form*

$$\rho_{12} = |\psi_1\rangle\langle\psi_1| \otimes |\psi_2\rangle\langle\psi_2|, \tag{2.6}$$

then it is called pure separable. If it can be expressed as

$$\rho_{12} = \sum_i p_i \rho_1^i \otimes \rho_2^i, \qquad \sum_i p_i = 1 \tag{2.7}$$

then it is mixed separable. Otherwise it is called entangled.

In the GME experiment the two masses are thus indeed entangled with each other provided $\Delta\phi_{LR} + \Delta\phi_{RL} \neq 2n\pi$, $n \in \mathbb{Z}$. The maximally entangled state in this case is reached for $\Delta\phi_{LR} + \Delta\phi_{RL} = \pi$.

2.1.1 Witnessing entanglement by correlating spatial with internal degree of freedom

To witness the entanglement after refocusing of the two masses, the spatial entanglement needs to be encoded in a distinct, non-spatial but internal degree of freedom. One way of implementing the protocol, as suggested by Bose et al [14], is to use two adjacent Stern-Gerlach interferometers by mapping the spatial entanglement to spin entanglement by using neutral atoms embedded with a spin degree of freedom. One particular branch may be then correlated as e.g. $|L\rangle \otimes |\uparrow\rangle$. This is done to allow for spatial splitting and refocusing via a magnetic field dependent on the spin $|\uparrow/\downarrow\rangle$. In this fashion, at the end of the refocusing stage[3], the joint system state is

$$|\psi(\tau)\rangle \propto \Big(|\uparrow\uparrow\rangle + e^{i\Delta\phi_{LR}}|\uparrow\downarrow\rangle + e^{i\Delta\phi_{RL}}|\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\Big)|C\rangle_1 |C\rangle_2 \tag{2.8}$$

with $|C\rangle_a$ being the central spatial position of test mass a. More explicitly, this may be achieved using micro-diamonds with nitrogen-vacancy (NV) centre spin [22, 23].

A way to witness this entanglement is by measuring an entanglement witness.

Definition 4 *An entanglement witness $\mathcal{W}$ of a bipartite state space $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is defined [24] to be a Hermitian observable such that its expectation value satisfies Tr$[\mathcal{W}\rho_{12}] \geq 0$ for any separable state ρ_{12}.*

While originally [14] an entanglement witness was proposed as

$$\mathcal{W}' = \mathbb{1} \otimes \mathbb{1} + X \otimes Z + Y \otimes Y, \tag{2.9}$$

Chevalier et al [18] suggested an *optimal entanglement witness*:

$$\mathcal{W} = \mathbb{1} \otimes \mathbb{1} - X \otimes X - Z \otimes Y - Y \otimes Z, \tag{2.10}$$

where $\{\mathbb{1}, X, Y, Z\}$ form the set of Pauli operators.

2.2 Optimisation of entanglement witnesses

Finding optimal entanglement witnesses to detect entanglement reliably, robustly and with minimal effort for the GME protocol (and especially more general setups to witness multipartite entanglement) is an area of active research [21, 25–31]. The optimal entanglement witness found by Chevalier [18] is given in Eq. 2.10. Its derivation is based on the Peres–Horodecki criterion or positive partial trace (PPT) criterion [32], which is often used as an entanglement quantifier for bipartite systems. More precisely, the system is entangled if one can show negative eigenvalues of the PPT density matrix.

As opposed to the initially found entanglement witness [14], the PPT entanglement witness shows entanglement for arbitrary interaction times τ given small enough decoherence rates. Thus, the experimental proposal is much less limited given the fact that increasing τ safely is difficult as decoherence becomes problematic during free-fall. This opens the window for robust entanglement detection despite even very small phase accumulation. This is particularly noteworthy, as one of the main challenges

[3] Again, the refocusing time scale is assumed to be short $t_f \approx \tau$, so no further phase is induced.

of the experiment is to isolate two massive particles and protect them from decoherence [10, 13–15, 33].

Furthermore, a more recent paper [34] provides a more general scheme to construct fidelity witnesses for gravitational entanglement, putting the previously proposed witnesses into context.

2.3 Does GME really imply quantum gravity?

One of the first people to conceive an experiment to demonstrate the quantum nature of gravity was Richard Feynman. In 1957 he suggested preparing a mass in a spatial superposition and detecting the quantum character of gravity by witnessing interference of the gravitational field [35]. However, as pointed out by Marletto and Vedral [15], this experiment is inconclusive in showing the entangling capacity of the gravitational field with the test mass, which is necessary to show that the effect of the gravitational phase induction could not have arisen from a classical field. While this thought experiment intended to show gravity is quantum is not sufficient, the original GME protocol, which was set up to finally be conclusive on this matter, is also contended in the literature. Simply observing the gravitational phase imprinted on the system does not imply that the mediator is quantum. We have to ask ourselves: Can gravity as a mediator be shown to be quantum if not only a phase is induced but also entanglement mediated as in the GME protocol?

2.3.1 Basic argument of impossibility of LOCC to generate entanglement

The main idea behind the GME experiment is related to a well-known result from quantum information theory that does not require knowing the specific form of the gravitational field, nor does it assume a specific model of quantum gravity. While many argue that the key to the experiment is the standard theorem that local operations and classical communications (LOCC) described by a mediator $\mathcal{F}$ cannot generate entanglement between two systems $\mathcal{Q}_1$ and $\mathcal{Q}_2$ [36, 37], this is not quite true and insufficient as gravity cannot be assumed to obey quantum theory as claimed by some[4]. The very precise formulation of the argument is thus very subtle and clearly of a foundational nature. Thus, Marletto and Vedral [15, 39] have correctly hinted in earlier papers that a more general approach is necessary. The proof presented in Ref. [40] does not assume the quantum nature of the mediator in some way or another a priori like in earlier arguments, but derives the result only from general information theory notions and locality

[4]However, there is some debate in the literature and some authors still insist on the sufficiency of this basic theorem [38].

using constructor theory [41].

Theorem 1 *Entanglement generation between two quantum systems $\mathcal{Q}_{1,2}$ through a local mediator $\mathcal{F}$ demonstrates the non-classicality of the mediator [40].*

Gravity is then, however, not necessarily quantum, but simply non-classical. Non-classicality in this context refers to the existence of complementary observables on the gravitational field. Therefore, if one assumes that the gravitational interaction between two masses is a local channel and hence is not classified as action-at-a-distance, then we can invoke the above very general information-theoretic considerations to certify that gravity cannot be a classical mediator if entanglement has been generated.

2.3.2 Objections to conclusions drawn from the experiment

First of all, the debate on the conclusions to be drawn from GME is quite active and given the different views on the matter, it may be too early to declare it settled. Due to the vast amount of literature on this topic, the aim of this dissertation is only to give an introduction to some of the counterarguments and explain why they arise. Most recently, it has been insisted upon by Martin-Martinez and Perche [42] that the entangling capacity of gravity does not imply non-classicality of gravity.

While the synergy of two different research communities meeting at the intersection of quantum information and quantum gravity is very exciting and fruitful, there is also often a potential for confusion. This is, in particular, the case regarding the notion of locality as it is (implicitly) understood. In quantum information, one refers to *system locality* as the principle of uncorrelated quantum operations between two systems as being separable [43]. The high-energy physics community works in a different framework, and refers to *event locality* as the principle of events in spacetime not affecting each other if they are causally disconnected [44]. The notions of locality and causality themselves are also often confused.

As pointed out by Ref. [42], there are inconsistencies in the argument of the impossibility of entanglement generation by LOCC, as the additional assumption is made that the gravitational field mediates through system locality. If a priori Quantum Field Theory (QFT) is used to describe gravity, then gravity will automatically satisfy system and event locality. However, the crucial point is that the a priori embedding of gravity into a quantum field theory framework is criticised as a circular argument. The requirement must be thus to consistently show that classical theory cannot have generated GME and must have been sourced by a non-classical gravitational field with non-classical features such as being in a superposition state. It is argued in Ref. [42] that if sufficient entanglement is generated within the light-crossing time between the spatial branches, then gravity can be inferred to be non-classical. For longer interaction times τ a classical field could have sourced the entanglement as well though. Regardless, it still claims that one could investigate how spatial superpositions of matter could give rise to gravitation.

As some authors have shown [38], there are mechanisms for spin-2 gravitons to entangle masses. However, it is contended that showing the existence of such a mechanism in the quantum field theory framework would not count as evidence for its actual validity of it in nature. The problem with this is that also local classical fields could be a possible explanation for GME [42, 45–48]. Similarly, Fragkos et al [49] argue that the quantisation of gravity cannot be inferred from the GME experiments, which would offer a plethora of ambiguous interpretations. Apart from inferring gravity to be a quantised mediator, one can also generate entanglement by other non-local means like in absorber theory [50]. These authors see locality as a feature that cannot be assumed a priori. This argument is advocated often with the reasoning that only a minimal amount of assumptions should be made on the nature of gravity. Given the last hundred years of unsuccessful attempts to find a theory of quantum gravity, radically new ways of thinking may be required.

2.3.3 Further vindication for the GME protocol

As argued in Ref. [51] on the other hand, in the philosophy of science additional knowledge *needs* to be assumed a priori. Given only one GME experiment, one can and should refer to what has been verified by other independent experiments. In this spirit, while there are potentially other theoretical classical explanations of gravity mediating entanglement, it is more plausible to interpret GME as evidence of a quantum superposition of geometry [52]. The GME experiment is thus claimed to be possibly the best attempt so far to reveal the entanglement of spacetime geometry with the massive particles interacting gravitationally.

While the original GME protocol [14] may not be able to certify non-classical behaviour of gravity on its own, it can at least rule out some classical theories of gravity [53]. For example, gravitational collapse models and quantum field theory in curved spacetime are shown incapable of entangling two spatially superposed masses [54]. Moreover, the original protocol [14] was based on an argument assuming (non-local) Newtonian gravitational interaction, which seems to disqualify it already in terms of its ability to certify the non-classicality of gravity [55]. However, as discussed by a refined gedankenexperiment in Ref. [56], GME through a Newtonian interaction may imply already entanglement with the graviton degrees of freedom.

In conclusion, the GME protocol remains a highly controversial object of debate regarding the conclusions drawn from it regarding the nature of gravity. There are both opponent as well as proponent arguments regarding the implications of GME experiments as indirect witnesses of the non-classicality of gravity.

2.4 Experimental implementation and challenges

Many different physical platforms to encode the entanglement into internal degrees of freedom of two massive objects have been identified. In the following we will outline the experimental platforms and abilities that have been suggested, first starting by Ref. [14] and then giving a more thorough report on the status quo.

In the original paper by Bose et al [14], neutral mesoscopic masses of mass $m = 10^{-14}kg$ are put into spatial gravitational cat states $\propto |L\rangle + |R\rangle$. The micro-crystals of radius $r = 1\mu m$ are assumed to be separated $d - \Delta x \approx 200\mu m$ at the closest approach, far enough to keep the interaction of the Casimir-Polder forces [57] at bay. The distance of closest approach refers here to the separation between the two adjacent branches corresponding to $|RL\rangle$. Note that $\Delta x = 2\delta$ is the distance between the spatial branches of each mass. The assumption is made that the branch at the closest approach is the dominant contribution to the system entanglement with $d - \Delta x \ll d, \Delta x$ and so $\phi_{RL} \gg \phi_{LR}, \phi$. With $d = 450\mu m$, the spin-dependent splitting is given by the expression

$$\Delta x \propto \frac{1}{2}\frac{g\mu_B \partial_x B}{m}\tau_i^2 \tag{2.11}$$

where $g = 2$ is the electron's gyromagnetic ratio, μ_B is the Bohr magneton and B is the inhomogeneous magnetic field causing the spatial splitting. For $\Delta x = 250\mu m$, Bose et al stipulate an interaction time $\tau = 2.5s$ for a sufficient phase accumulation at the closest approach

$$\Delta\phi_{RL} = \phi_{RL} - \phi = G\frac{m^2}{\hbar}\tau\left(\frac{1}{d - \Delta x} - \frac{1}{d}\right) \tag{2.12}$$

which amounts to $\Delta\phi_{RL} = 0.44$.

Even though advances have been made in terms of realising the creation of spatial superposition states for such large massive objects [22, 58–70] through many different techniques, current experimental capabilities still lack behind with e.g. macromolecules with mass $m = 10^{-22}kg$ and splitting separation $\Delta x = 0.25\mu m$ [59]. Apart from micro-diamonds with NV centre spin, other physical platforms have been discussed. These include work on massive molecules, Bose-condensates, or nano-mechanical oscillators [15]. Furthermore, it needs to be noted that for any valuable and statistically significant demonstration of entanglement generation due to gravity, the protocol needs to be repeated many times, each time requiring the creation or recycling [71] of neutral test masses, suitable for the experiment.

Creating and maintaining two such massive objects in spatial superposition is already quite difficult for sufficient phase generation. Keeping them apart for relatively large distances on the order of hundreds of microns is a further tremendous experimental challenge. The non-negligible presence of Casimir-Polder interactions at short distances thus makes the wish of maximising the dominant phase induced on the $|RL\rangle$ state $\Delta\phi_{RL}$ at the closest approach significantly more challenging. The Casimir interaction energy [57] contribution is

$$E_{Casimir} = \frac{23\hbar c R^6}{4\pi(d-\Delta x)^7}\left(\frac{\epsilon-1}{\epsilon+2}\right)^2 \tag{2.13}$$

where ϵ is the dielectric constant of the two massive objects. As investigated in Ref. [72], the effect of the Casimir potential can be reduced by inserting a thin conducting plate between the two matter-wave interferometers. This allows for the screening of electromagnetic dipole-dipole interactions as well as Casimir interactions. Hence, the distance of closest approach can be minimised with suggested particle masses of 10^{-15}kg. It is claimed that the entanglement phase $0.01 rad$ for $\tau = 1s$ is still detectable and the decoherence can be mitigated as well.

As highlighted by Chevalier et al [18], sources of potential entanglement generation other than gravity do not necessarily need to be eliminated at all cost. Their contributions can simply be distinguished from the effect due to gravity by a statistical analysis a posteriori. Entanglement verification [73] provides the basis for conducting likelihood ratio tests that aim at disproving the null hypothesis of the absence of gravity as a contribution to the induced phase [18] given Casimir-Polder interaction. As calculations [18] indicate, one can already make a strong case for ruling out gravitational contribution to entanglement generation for 10^2 to 10^3 repetitions of the protocol. Quantum state tomography [74, 75] was also shown to be able to estimate an entanglement monotone and discriminate different entanglement generation contributions even from interactions with unknown coupling strengths [18]. Thus, this gives further confidence in the results regardless of van-der-Waals forces, other dipole forces, or signatures of a fifth force being present, for example. Nevertheless, a proper statistical analysis can be guaranteed only through easy repeatability of the experiment.

2.5 Extensions and variations of GME protocol

Aside from the advances in terms of the original GME protocol using two massive objects, Tilly et al [26] also considered a generalisation of the protocol. They examined various geometrical setups and generalised the number of states in the spatial superposition. They demonstrated that under the presence of decoherence multi-dimensional qudit setups can improve the entanglement generation compared to the two-qubit setup. The PPT entanglement witness in Eq. 2.10 is still optimal for different GME protocol setups involving massive qutrits and qudits [25, 26]. In general, not all entangled qudit states are determined by the PPT witness generalised to qudits [32, 76].

The gravitational interaction was also shown to be enhanced through a massive mediator between the two massive test particles in the GME protocol [33]. Also, alternative protocols have been proposed that are supposed to testify to the non-classicality of gravity through other means than entanglement generation. This includes, for example, non-gaussianity [77] which is impossible by classical theories of gravity. Further work in the subject area included mechanistic descriptions of GME through virtual graviton exchange [38] as well as explicit quantum field descriptions revealing the superposition of geometry of the gravitational field [52].

As elaborated on in Section 2.3 there are still a lot of discussions at the time of writing, however, on the actual significance of the proposal to witness quantum gravity due to gravity-mediated mediated entanglement protocols. Regardless, the GME protocol certainly has been very influential in elevating the formerly rather niche table-top quantum gravity field into a research field of its own. Hence, many aspects and further extensions have been investigated, see Section 2.5. In the next Section, we will now build upon the existing protocol and examine the use of light pulses for the experiment.

Chapter 3

Photonic GME protocol

3.1 Motivation

The original GME protocol suffers from several experimental challenges and limitations, some of which can be overcome with sufficient care, but others are intrinsic constraints. First, the need for cat states of large masses is difficult to accommodate for in a repeatable fashion for good statistical significance for witnessing entanglement. Secondly, while it can be dealt with by likelihood tests or quantum state tomography, the presence of residual interactions such as the Casimir interaction may also contribute to entanglement generation, which thus may distort the conclusions drawn from the effect of gravity. Thirdly, the distance of closest approach between the masses is limited[1] to be well beyond the diffraction limit to keep the non-overlapping position eigenstate approximation.

To resolve some of the challenges faced by the traditional GME protocol using massive objects, this paper suggests the use of light beams as an alternative physical platform to realise GME. This is motivated by the fact that light, according to general relativity, also sources the gravitational field, a result first investigated more closely in a seminal paper by Tolman et al [16] on the gravitational field produced by a *thin pencil of light*. Since then many other works on this topic have been done [78–82], yet without any empirical confirmation. While light may not solve all the challenges and comes with different difficulties, this work may also be seen independent of the GME protocol as a pathway to demonstrate the (very small) gravitational coupling of light beams experimentally in the not-too-distant future.

In addition, modern laser technology [83, 84] enables us to tune and control high intensity light beams each containing a multiplicity of photons with an unprecedented capacity of empirical repeatability. While each photon may only have a negligibly small effect on gravity mediated entanglement due to its small coupling to gravity, the collec-

[1] Again, the Casimir interaction becomes also stronger at a close distances.

tive effect and reliability of light as a source of entanglement generation may outscore any massive counterpart as a platform for witnessing gravity mediated entanglement. While some of the experimental demands in the GME protocol are not yet possible, the high pace of technological innovation in the industry may make the protocol feasible soon.

Due to the bosonic nature of the physical constituents of light beams mediating entanglement as opposed to fermionic matter particles, the separation between light beams may be tuned arbitrarily small[2] with inclusion of diffraction effects [85]. The ability to bring the pulses close together, as there is virtually no closest approach limit, may help compensate for the weakness of the gravitational field sourced by light as opposed to matter. Light pulses thus favour potentially higher entanglement generation and more accurate calculations without the need of resorting to highly restrictive approximations.

In addition, the photonic GME protocol is devoid of any undesirable interactions other than gravity and hence the statistical difficulty is shifted from systematic errors being present in the form of other interactions for massive objects to merely a need for improved sensitivity for small entanglement generation. Thus, we have shifted the problem from discriminating between different relevant and competitive sources to discriminating between background and signal - which may not be a problem at all. As said before, even if the entanglement generation - if the model is accurate - is less strong than in the massive case, there is certainly a case for a consideration of these results in light of potentially new experimental and technological advances in the near future.

Furthermore, as argued in Ref. [51], the original experiment with massive objects provides only a playground to test the non-relativistic regime of gravity but fails to recognise any potential features of gravity as a relativistic quantum field. Now using light already provides a fully natural framework for investigating both relativistic and quantum effects of testing quantum gravity. Due to the easier tunability of frequency (as opposed to mass), the present protocol may also be used to demonstrate evidence on finer features of gravity and other deviations from classicality.

To date, the main reason why light was not considered for the protocol is due to the extremely small single photon-photon coupling through gravity predicted by current quantum field theory models. To our knowledge, the gravitational coupling of two light pulses has not yet been experimentally verified. As stated before, there is still some debate on the capacity of the protocol to show that gravity is quantum or non-classical. Even if the photonic GME protocol cannot be used to certify that gravity is non-classical as claimed, it can still be envisioned as a way of indirectly witnessing for

[2] This is possible if decoherence and diffraction can be taken into account. Furthermore, the experiment still needs to distinguish between the pulses at close distance.

the first time the gravitational coupling of light. Measuring entanglement generation as an indirect means is much less ambitious than trying to measure directly the gravitational deflection of two light pulses. This would be the first step to investigate gravitational light-light interactions experimentally, an investigation that would have been doomed utterly hopeless by some research communities not too long ago and would emancipate this protocol from the original desire to demonstrate the non-classicality of gravity.

3.2 Preliminary considerations on feasibility

Before we embark on establishing the full formalism in quantitatively showing the entanglement capacity of the photonic GME protocol, what follows is a somewhat simplistic approach to estimate the feasibility of the protocol. It should only serve as a back-of-the-envelope calculation taken with a pinch of salt.

While photons interact with each other gravitationally, the interaction of two massless particles cannot be Newtonian, but the adequate description must include general relativity. While photons are massless, a general relativistic calculation reveals that the acceleration of antiparallel light beams is twice as much as calculated from the Newtonian theory of gravitation with an effective "rest" mass $m = \frac{E}{c^2}$, where E is the energy of the beam. Hence, we will invoke this notion of photons as massive corpuscles for a reasonably accurate order-of-magnitude calculation of phase induction.

We assume a non-zero photon rest mass m and speed of photons $v < c$, where c is the speed of light in vacuum. This might be achieved by photons propagating through a medium, the gravitational interaction between two photons is simply assumed to be the classical gravitational potential energy $U(r) = -\frac{Gm^2}{r}$, where r is their separation[3]. Using de Broglie relations $E = mc^2$ and $E = \frac{hv}{\lambda}$, and taking now m to be the mass of one pulse containing N particles, we obtain

$$m = \frac{hvN}{c^2\lambda}, \tag{3.1}$$

where λ is the wavelength of the photons. The interaction potential then takes on the form

$$U(r) = -\frac{Gh^2v^2N^2}{c^4\lambda^2 r}. \tag{3.2}$$

We simplify the calculation by assuming co-propagation at constant separation as in the original protocol geometry, which can be ensured by an embedding into a medium.

[3]Here we have assumed photons in non-overlapping position eigenstates as in the original GME protocol. In particular, this approach breaks down if we let $r \to 0$ and is completely invalid.

Due to the non-zero gravitational interaction in a medium for co-propagating light pulses [79] at velocities smaller than the speed of light (e.g. in a waveguide), we can proceed further to look into the order of magnitude of phase ϕ accumulated for an interaction time t and distance r:

$$\phi = \frac{Ut}{\hbar} := -const \cdot \frac{tv^2N^2}{r\lambda^2}, \qquad const = \frac{2\pi Gh}{c^4} = 3.43 \cdot 10^{-77}\mathrm{m}^1\mathrm{s}^1. \tag{3.3}$$

The relevant (dominant) phase at the closest approach in the state in Eq. 2.3 is $\Delta\phi_{RL} = \phi_{RL} - \phi$ (see Eq. 2.12). For a laser with $N = 10^{20}$ photons, $\nu = \frac{v}{\lambda} = 10^{15}s^{-1}$ as $v \approx 10^8 m/s$ and $\lambda \approx 10^{-7}m$, we must require the distance at the closest approach to be larger than the wavelength, i.e. $d - \Delta x > \lambda$. If we assume that the wavelength has the same value as in vacuum at the speed of light, we arrive at $d - \Delta x > \frac{c}{\nu} = 3 \cdot 10^{-7}$m. To have a sufficiently large phase and be consistent with the diffraction limit at which this massive corpuscle picture would be even less accurate, we may choose $d = 2 \cdot 10^{-6}$m and $\Delta x = 10^{-6}m$. Then $\frac{tv^2N^2}{\lambda^2}(\frac{1}{d-\Delta x} - \frac{1}{d}) = t \cdot 5 \cdot 10^{75}\mathrm{m}^{-1}s^{-2}$ and hence

$$|\Delta\phi_{RL}| = (3.43 \cdot 10^{-77}) \cdot (5 \cdot 10^{75}) \cdot t = 1.71 \cdot 10^{-1} \cdot t \tag{3.4}$$

By having particles interact (e.g. in loops) we can be content with a phase of a $|\Delta\phi_{RL}| \approx 1mrad = 10^{-3} \cdot \frac{180}{\pi}$, which makes it possible to pick up after some repetitions and data analysis. It corresponds to a time of $\tau = 0.33s$.

While this is quite a long time for light travel, the quadratic dependence on the photon number N is quite promising, as technological progress in lasers will quickly enhance the experimental feasibility. Thus, this preliminary result is suggestive of a further, more technical investigation into the photonic protocol as a feasible alternative to the conventional GME protocol.

3.3 Light field preparation

3.3.1 Description of basic setup

Research regarding entangled states of photons is motivated partially by their widespread use in quantum technologies, quantum communication, computation, and metrology [86–88]. One of the main difficulties in the massive GME protocol is the creation of massive spatial superposition states. A superposition $\propto |L\rangle + |R\rangle$ of two distinct states $|L\rangle$ and $|R\rangle$ (with $\langle L|R\rangle = 0$) is known as a Schrödinger cat state. For the experiment, it is crucial to create two light pulses each in a spatial superposition. That way they will each pick up some gravitational phase. If each of the two light pulses is put into any other non-spatial optical Schrödinger cat state, then no gravitational entanglement can be witnessed. As the separation between the two branches of the first pulse is equal with respect to the other two branches of the second pulse, there is - classically - no distinct gravitational phase generation between each branch. If the gravitational coupling depends not only on the spatial mode but also on another internal mode such as polarisation, an internal degree-of-freedom-dependent phase induction and thus entanglement generation can be ensured. However, to date, no experiment has been conducted so far [89] that would show the coupling of gravity to any internal degree of freedom of a system such as polarisation. Thus, the spatial position needs to be correlated with some internal degree of freedom. While in the massive GME experiment the spatial path was correlated to a spin degree of freedom, the quantum optical analogue is correlating the spatial path with the light beam's polarisation degree of freedom[4]. This is to ensure that the gravitational phase is not erased when the two light pulses are refocused again.

Let us consider two Mach-Zehnder interferometers, as shown in Fig. 3.1, that receive two counter-propagating light pulses as inputs. The length of the interferometer is D and the transverse separation of the two centres is d. Each input pulse enters through a polarising beam splitter. The reason we have chosen two counter-propagating light pulses is that no gravitational phase can be induced in vacuum for two non-overlapping light pulses [17, 79].

After passing the light pulses through the beam splitter separating light into horizontal or vertical polarisation, a quarter-wave plate may be used to correspondingly have right-handed and left-handed circular components in the spatial branches separated. At the end of the gravitational phase induction, the light pulses pass again through quarter wave plates before passing a beam splitter to detect the entanglement.

3.3.2 Mathematics of beam splitters recapped

Operationally, we may consider the beam splitting operator $\hat{B}$ as acting upon a tensor product state of the two input fields of the form $|\cdot\rangle_x|\cdot\rangle_z$, where the index x, z refers to the propagation direction of the input fields (and thus also the different polarisation modes). The beam splitting operator $\hat{B}$ can then be expressed [91] as

$$\hat{B} = \exp\left(\frac{\theta}{2}(\hat{a}_x^\dagger \hat{a}_z + \hat{a}_x \hat{a}_z^\dagger)\right), \tag{3.5}$$

where the respective creation and annihilation operators for each input mode are given by $\hat{a}_{x,z}^\dagger$ and $\hat{a}_{x,z}$. Note that the angle θ is related to the transmittivity and reflectivity coefficients $t = \cos(\theta/2)$ and $r = \sin(\theta/2)$ that are normalised $|t|^2 + |r|^2 = 1$ to satisfy energy conservation.

[4] Apart from polarisation entanglement, optical mode entanglement through e.g. Dicke superradiance may also be an option to consider [90] in the future for the GME experiment.

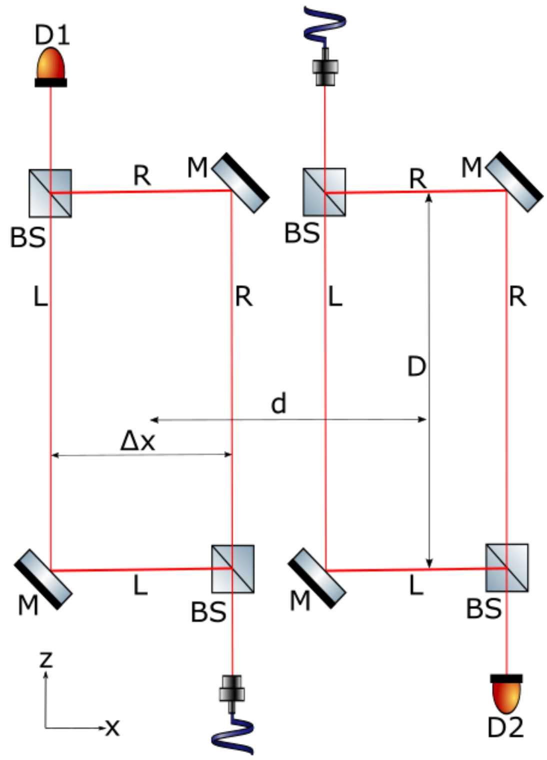


Figure 3.1: The Mach-Zehnder interferometer setup of the photonic GME protocol. Two counter-propagating light pulses of length L traverse a beam splitter upon their entry into the interferometer, which entangles their polarisation degree of freedom with their spatial path. With the potential insertion of quarter waveplates each beam splits into left- and right-handed circular polarisation. The two light pulses interact gravitationally. At the end of the protocol, each spatial branch coincides again meeting at the second beam splitter that disentangles (after the use of wave plates) the polarisation from the spatial mode. The entanglement is then witnessed by local polarisation measurements.

The input operators $\hat{a}_i$ are related to the respective output operators $\hat{b}_i$ in the Heisenberg picture via

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \hat{B} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} \hat{B}^\dagger \tag{3.6}$$

where the Baker-Hausdorff theorem [19, 92] has been invoked and leads more explicitly to

$$\hat{b}_x = \hat{B}\hat{a}_x\hat{B}^\dagger = \cos\left(\frac{\theta}{2}\right)\hat{a}_x + \sin\left(\frac{\theta}{2}\right)\hat{a}_z \tag{3.7}$$

$$\hat{b}_z = \hat{B}\hat{a}_z\hat{B}^\dagger = \cos\left(\frac{\theta}{2}\right)\hat{a}_z - \sin\left(\frac{\theta}{2}\right)\hat{a}_x \tag{3.8}$$

Per definition, each beam splitter is assumed to transmit and reflect equally with no losses in our setup. We can enforce the requirement for a 50:50 beam splitter by setting the phase $\theta = \pi/2$ leading to:

$$\hat{b}_x^\dagger = \hat{B}\hat{a}_x^\dagger\hat{B}^\dagger = \frac{1}{\sqrt{2}}(\hat{a}_x^\dagger + \hat{a}_z^\dagger) \tag{3.9}$$

$$\hat{b}_z^\dagger = \hat{B}\hat{a}_z^\dagger\hat{B}^\dagger = \frac{-1}{\sqrt{2}}(\hat{a}_x^\dagger - \hat{a}_z^\dagger) \tag{3.10}$$

using the Baker-Campbell formula [19].

Eventually, after having successfully put each of the two light pulses in a spatial superposition correlated with its polarisation degree of freedom, each branch will pick up a gravity mediated phase, as elaborated on in Section 3.5.3. The gravitational interaction prior to the first light pulse impinging on the mirror is not considered as the time is considered negligible[5]. Through the action of appropriate waveplates, one can always ensure that the four different branches are all described by either left or right-handed circular polarisation, for which our derivation in Section 3.5 can be used. For example, circularly polarised light can be produced by passing linearly polarised light through a quarter waveplate (and vice versa) [93].

At the end of the gravitational interaction phase, the joint spatial-polarisation subsystem state will have evolved to

$$\begin{aligned} |\psi(\tau)\rangle \propto\, & e^{i\phi_{LL}(\tau)}|L;\circlearrowleft\rangle_1|L;\circlearrowright\rangle_2 + e^{i\phi_{LR}(\tau)}|L;\circlearrowleft\rangle_1|L;\circlearrowleft\rangle_2 \\ & + e^{i\phi_{RL}(\tau)}|R;\circlearrowright\rangle_1|L;\circlearrowright\rangle_2 + e^{i\phi_{RR}(\tau)}|R;\circlearrowright\rangle_1|R;\circlearrowleft\rangle_2 \end{aligned} \tag{3.11}$$

3.3.3 Why coherent states cannot be used

Even though light beams in coherent states [94–97] contain a high number of photons desirable to source a sufficient metric perturbation, they cannot be used for the GME protocol. We will now show that if the input state incident on the beam splitter is a vacuum-coherent state of the form $|\psi\rangle_{in} = |0\rangle_x \otimes |\alpha\rangle_z$, where

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{N=0}^{\infty}\frac{(\alpha)^N}{\sqrt{N!}}|N\rangle \tag{3.12}$$

then no entanglement can be generated [92]. With the action of the displacement operator

$$\hat{D}_z(\alpha) = \exp(\alpha\hat{a}_z^\dagger - \alpha^*\hat{a}_z) \tag{3.13}$$

on the initial state, i.e. $|\psi\rangle_{in} = \hat{D}_z(\alpha)|00\rangle = \exp(\alpha\hat{a}_z^\dagger - \alpha^*\hat{a}_z)|00\rangle$, we can express the output state after the operation of the beam splitter by generalising the above relations

[5]This assumption is similar to assuming a negligible time for splitting and refocusing in the massive GME protocol case.

on the action of $\hat{B}$ on $\hat{a}_i$ as

$$|\psi\rangle_{out} = \hat{B}|\psi\rangle_{in} = |\frac{\alpha}{\sqrt{2}}\rangle_x \, |\frac{-\alpha}{\sqrt{2}}\rangle_z \, . \tag{3.14}$$

Thus, this completes the proof that a one-mode coherent state emerges as two coherent states not entangled with each other. As this is not in the desired form, no coherent states can be used for the protocol.

3.3.4 How can the beamsplitter entangle two spatial paths?

As we have seen, a vacuum-coherent state does not become entangled by the beam splitter operation. The maximally entangled state after beam splitting in each interferometer is

$$|\psi\rangle_{out} = \frac{1}{\sqrt{2}}(|N\rangle_x|0\rangle_z + |0\rangle_x|N\rangle_z) \tag{3.15}$$

which is also called the *NOON* state. This is the ideal cat state that one wants to achieve in the GME protocol.

Due to the high interest for use in quantum information science, several protocols have been proposed to generate NOON states [98–103]. Unfortunately, high-NOON states currently contain much less photons than necessary to source a sufficient metric perturbation with $N < 10$ photons. However, we may assume that in the future high NOON states can be realised. The approach presented by Ref. [104] to generate high NOON states is, in principle, scalable up to arbitrarily high photon numbers. One advantage of NOON states is that for N photons, the phase induced between the two paths increases N times as fast as for classical light [92] making them much more sensitive to gravitational phase induction as calculated in Section 3.5.3.

The natural question arises: Which input states can be used to obtain an entangled two-mode state subsequent to the beam splitter action? This question was investigated by Kim et al and Asboth et al [105, 106] showing the necessity and sufficiency of a non-classical state in one input mode.

Approximate NOON states may also be generated by considering two-mode squeezed vacuum states as inputs. We first define the two-mode squeeze operator [92]

$$\hat{S}_2(\xi) = \exp\left(\xi^* \hat{a}_x \hat{a}_z - \xi \hat{a}_x^\dagger \hat{a}_z^\dagger\right) \tag{3.16}$$

where $\xi = re^{i\theta}$ is the squeezing parameter. The two-mode squeezed vacuum state $|\xi\rangle_2 = \hat{S}_2(\xi)|0,0\rangle$ is then found in Ref. [107] to give

$$|\xi\rangle_2 = (1-|\xi|^2)^{\frac{1}{4}} \sum_{N=0}^{\infty} \frac{\sqrt{(2N)!}}{N!}(-\frac{\xi}{2})^N |2N\rangle, \qquad |\xi|< 1 \tag{3.17}$$

The action of the beam splitter $\hat{B}$ then yields up to a normalisation constant for even and odd photon number states

$$|\xi_{2N}\rangle \propto \sum_{k=0}^{\infty} \frac{1}{\sqrt{(2k)!}} \frac{(2k+2N)!}{(k+N)!} (-\frac{\xi}{2})^{k+N} |2k\rangle \tag{3.18}$$

$$|\xi_{2N+1}\rangle \propto \sum_{k=0}^{\infty} \frac{1}{\sqrt{(2k+1)!}} \frac{(2k+2N+2)!}{(k+N+1)!} (-\frac{\xi}{2})^{k+N+1} |2k+1\rangle \tag{3.19}$$

Having entangled the two spatial paths with each other by correlating polarisation degree of freedom to the spatial path, further research may be done to specify in more detail the feasibility of the scheme. Admittedly, substantial technological advances may be required.

3.4 Lorentz covariant phase generation through path integrals

The original GME protocol assumed Newtonian gravitational action-at-a-distance, but to make the locality of the gravitational field present, a manifestly Lorentz covariant approach using path integrals [108] known from quantum field theory was first invoked by Brukner et al [109]. We will adopt this formalism due to its freedom of choice in inserting the desired ingredients coupling to each other gravitationally via the action.

As explained previously, the GME protocol consists of systems $\mathcal{Q}_a$ (where $a = 1, 2$), that are both described by their spatial motion $\vec{x}_a^{s_a}(t)$, which depends on an (unchangeable, fixed) internal degree of freedom s_a, which may take on one of two values to allow for a particle to be put into a superposition state. In the massive case, this internal degree of freedom may correspond to the spin of the particle whereas for light, it corresponds to its polarisation state. As explained earlier, $|\circlearrowleft / \circlearrowright\rangle$ denotes left/right-handed circular polarisation. The internal configuration between two of the four different branches is then described by a state $|\sigma\rangle = \bigotimes_a |s_a\rangle$.

By assuming weak gravity in the linear regime, one can consider the gravitational field $\mathcal{F}$ to be coupling to the systems $\mathcal{Q}_a$ and mediating the entanglement. If B is the field that causes the internal degree of freedom dependent splitting and refocusing of paths, then we assume its coupling with $\mathcal{F}$ and backreaction of s_a on B and of $\mathcal{F}$ on $\vec{x}_a^{s_a}(t)$ to be negligible. For the photonic setup, this means that the beam splitter $\hat{B}$ does not couple to the gravitational field itself and the beam splitter action only correlates the spatial motion with the polarisation state.

Given that the internal degree of freedom is correlated with the spatial path of each branch, the time evolution operators can be expressed as

$$U_{i\to f} = \sum_{\sigma} |\sigma\rangle\langle\sigma| \otimes U^{\sigma}_{i\to f} \tag{3.20}$$

where $U^{\sigma}_{i\to f}$ defined from initial and final total states[6] $|\phi^{i,f}\rangle = |\mathcal{F}^{i,f}[x_a^{i,f}]\rangle \otimes |x_a^{i,f}\rangle$. Using $\int \mathcal{D}\phi' = \int \mathcal{D}\mathcal{F}'\mathcal{D}x'$ and $U^{\sigma}_{i\to f}|\phi^i\rangle \equiv |\phi^f\rangle$ by definition[7], we find

$$U^{\sigma}_{i\to f} = \int_i^f \mathcal{D}\mathcal{F}'\mathcal{D}x' \exp\left(\frac{iS[x'_a, \mathcal{F}[x'_a]]}{\hbar}\right)|\phi^f\rangle\langle\phi^i| \tag{3.21}$$

Using the stationary phase approximation (neglecting loop corrections), we get up to a normalisation constant

$$U^{\sigma}_{i\to f} \propto \int_i^f \mathcal{D}x' \exp\left(\frac{iS[x'_a, \mathcal{F}[x'_a]]}{\hbar}\right)|\phi^f\rangle\langle\phi^i| \tag{3.22}$$

where $x_a^{s_a}$ is the classical path. By the second stationary phase approximation and assuming orthogonality of the spatial states of the different paths we arrive at

$$U^{\sigma}_{i\to f} \propto \exp\left(\frac{iS^{\sigma}[x_a^{s_a}, \mathcal{F}[x_a^{s_a}]]}{\hbar}\right)|\phi^f\rangle\langle\phi^i| \tag{3.23}$$

where S^{σ} is the joint on-shell action.

Furthermore, if there exists a decomposition of the action into $S = S_0 + S_{\mathcal{F}}$ with S_0 corresponding to the action of the system Q_a and a function of x, B, σ, then S_0 serves as a global phase coincident for all internal degree of freedom configurations σ. The phase dependent on field mediation $S_{\mathcal{F}}$ can then be expressed as:

$$\phi_{\sigma} = \frac{S^{\sigma}_{\mathcal{F}}[x_a^{s_a}, \mathcal{F}[x_a^{s_a}]]}{\hbar} \tag{3.24}$$

Given the total initial state $|\Psi^i\rangle \propto |\phi^i\rangle \otimes \sum_{\sigma} A_{\sigma}|\sigma\rangle$, the final state can be written

$$|\Psi^f\rangle = U_{i\to f}|\Psi^i\rangle \propto |\phi^f\rangle \otimes \sum_{\sigma} A_{\sigma} e^{i\phi_{\sigma}}|\sigma\rangle, \tag{3.25}$$

which makes the entanglement generation through the mediating gravitational field $\mathcal{F}$ between the internal degrees of freedom at the end of the protocol explicit.
The entanglement witness in Eq. 2.10 was found to be optimal for the two-qubit setup for the massive case. Similarly, we will use it for the GME protocol with two light pulses to get:

$$\langle\mathcal{W}\rangle = \langle\Psi^f|\mathcal{W}|\Psi^f\rangle = \sum_{\sigma,\sigma'} A_{\sigma}A^*_{\sigma'} e^{i(\phi_{\sigma}-\phi_{\sigma'})}\langle\sigma'|\mathcal{W}|\sigma\rangle \tag{3.26}$$

[6]Here we have assumed that the field and the systems are not entangled with each other at the beginning and end of the protocol.
[7]see more on schematic notation for path integral measure [110]

3.5 Action for linearised gravity

The gravitational field sourced by systems suitable for the GME protocol can be described by a weak perturbation of the Minkowski background metric $\eta_{\mu\nu}$. In the following, the action for such a linearised theory of gravity may be derived using standard general relativity [111]. Weak gravitational fields $\mathcal{F}$, such as those weakly sourced by high-intensity laser beams, are described by a metric[8]:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}, \qquad h_{\mu\nu} \ll 1 \tag{3.27}$$

where $h^{\mu\nu} = \eta^{\mu\rho}\eta^{\nu\sigma}h_{\rho\sigma}$. The linearised Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{3.28}$$

where $T_{\mu\nu}$ is the energy-momentum tensor to be specified, follow [109] from the action

$$S_h = \frac{c^4}{64\pi G}\int d^4x\left(-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial^\mu h\partial_\mu h\right) + \frac{1}{2}\int d^4x h_{\mu\nu}T^{\mu\nu} \tag{3.29}$$

given in Lorenz gauge $\partial^\nu \bar{h}_{\mu\nu} = 0$, where we denote the trace reversal of a tensor $h_{\mu\nu}$ by $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$ and $h = \eta_{\mu\nu}h^{\mu\nu}$ as the trace of $h_{\mu\nu}$. With the Euler-Lagrange equations for $h_{\mu\nu}$, we get

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\bar{T}_{\mu\nu} \tag{3.30}$$

Thus, the on-shell action, after integration by parts, takes the form

$$S_h = \frac{1}{4}\int d^4x h_{\mu\nu}T^{\mu\nu} \tag{3.31}$$

The retarded solution [17, 109] of the wave equation in Eq. 3.30 is

$$h_{\mu\nu}(t,\vec{x}) = \frac{4G}{c^4}\int dx'^3 \frac{\bar{T}_{\mu\nu}(\vec{x}', t_r)}{|\vec{x} - \vec{x}'|} = \frac{4G}{c^4}\int dx'^3 \eta_{\mu\alpha}\eta_{\nu\beta}\frac{\bar{T}^{\alpha\beta}(\vec{x}', t_r)}{|\vec{x} - \vec{x}'|} \tag{3.32}$$

with the retarded time $t_r = t_r(t, \vec{x}, \vec{x}')$ defined by

$$ct_r = ct - |\vec{x} - \vec{x}'|. \tag{3.33}$$

3.5.1 Stress-energy tensor and metric perturbation for a single light pulse

The calculation follows closely a paper investigating the effect of the gravitational field sourced by a monochromatic laser pulse [17] on another test pulse, in which diffraction and overlap are disregarded. Later on, we want to extend these calculations by

[8] Metric signature: $(-,+,+,+)$

considering the effect of two counter-propagating (but shifted) laser pulses each of length L, and negligible extension in the xy-plane. The two laser pulses are separated by a distance D along the z-axis and by a distance $x_2 - x_1$ along the x-axis, where x_i denote the respective x-coordinates. Since the stress-energy tensor adds linearly, i.e. $T^{\mu\nu} = \sum_{a=}^{1} T_a^{\mu\nu}$, we will first consider the stress-energy tensor due to the presence of a single pulse.

The energy-momentum tensor $T_{\mu\nu}$ including the emitter and absorber, which need to be included in the calculation to satisfy energy-momentum conservation, can be decomposed into its contribution due to the pure pulse, the emitter and the absorber: $T_{\mu\nu} = T^p_{\mu\nu} + T^e_{\mu\nu} + T^a_{\mu\nu}$. In free space and flat space-time [112] we can express $T^{\mu\nu}$ using the field strength tensor $F_{\mu\nu}$:

$$T^{\mu\nu} = \frac{1}{\mu_0}\left(F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right), \tag{3.34}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$H/m is the vacuum permeability. By assuming an electromagnetic plane wave propagating in vacuum in the $\hat{z}$-direction, the corresponding energy-momentum tensor depends only on the combination $ct - z$. Electric and magnetic fields $\vec{E}$ and $\vec{B}$ are orthogonal to the direction of propagation, and thus, as stated in the seminal paper by Rätzel et al [17], the only non-vanishing components of the stress-energy tensor are given by $T^{00}, T^{zz}, T^{0z}, T^{z0}$. To show this, we know that as $\vec{E}, \vec{B} \perp \hat{z}$ and $\vec{E} \perp \vec{B}$, we may choose $\vec{E} = E\hat{x}, \vec{B} = B\hat{y}$. Then, we know $B = \frac{E}{c}$ from the Maxwell equations and the Poynting vector is $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \frac{EB}{\mu_0}\hat{z} = \frac{E^2}{c\mu_0}\hat{z}$.

The energy-momentum tensor in explicit matrix form is

$$T^{\mu\nu} = \begin{pmatrix} u & \frac{S_x}{c} & \frac{S_y}{c} & \frac{S_z}{c} \\ \frac{S_x}{c} & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ \frac{S_y}{c} & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ \frac{S_z}{c} & -\sigma_{zx} & -\sigma_{zy} & -\sigma_{zz} \end{pmatrix}, \tag{3.35}$$

where the electromagnetic energy density is

$$u = \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2) = \varepsilon_0 E^2 \tag{3.36}$$

and the Maxwell stress tensor is

$$\sigma_{ij} = \varepsilon_0 E_i E_j + \frac{1}{\mu_0}B_i B_j - \frac{1}{2}(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2)\delta_{ij}. \tag{3.37}$$

The only non-zero components of the stress-energy tensor are thus $T^{00} = T^{0z} = T^{z0} = T^{zz} = u$ with the only non-vanishing components of the metric perturbation for a single pulse $h^p_{00} = h^p_{zz} = -h^p_{0z} = -h^p_{z0} = h^p$. Thus, also the energy density for a pulse propagating in $+\hat{z}$ direction is a function of the combination $ct - z$ only. Due to the specified form of the pulse, we know that the non-vanishing components of the stress-energy tensor are $T_{\mu\nu} = Au(ct-z)\delta(x)\delta(y)$, where A is the effective area of the pulse in the xy-plane.

Consequently, evaluating the 00-component of Eq. 3.32 yields

$$h_{00}(t,\vec{x}) = h^p = \frac{4GA}{c^4}\int dx'^3 \frac{u(ct_r(\vec{x}',\vec{x},t) - z')\delta(x')\delta(y')}{|\vec{x} - \vec{x}'|} = \frac{4GA}{c^4}\int dz' \frac{u(ct_r(x,y,z,t,z') - z')}{\sqrt{\rho(x,y)^2 + (z'-z)^2}} \tag{3.38}$$

where we have denoted the retarded time $t_r = t - \sqrt{\rho(x,y)^2 + (z-z')^2}/c$ with

$$\rho(x,y) = \sqrt{x^2 + y^2}. \tag{3.39}$$

Choosing the variable substitution

$$\zeta(x,y,z,z') = (z'-z) + \sqrt{\rho(x,y)^2 + (z'-z)^2} \tag{3.40}$$

we arrive at

$$h^p = \frac{4GA}{c^4}\int_{\zeta(x,y,z,a)}^{\zeta(x,y,z,b)} d\zeta \frac{u(ct - \zeta - z)}{\zeta} \tag{3.41}$$

The integration boundaries a, b of each pulse contribution are chosen from the intersection of the world sheet boundaries of each pulse with the past light cone $\mathcal{J}^-$ of an observer located at spacetime position $x^\mu = (x,y,z,t)$ [17]. For pulses of length L, one defines the auxiliary integration boundaries $\bar{a}, \bar{b}$ as solutions of $t_r(z') = \frac{z'+L}{c}$ and $t_r(z') = \frac{z'}{c}$, which are given by

$$\bar{a}(x,y,z,t) = z + \frac{(ct-L-z)^2 - \rho(x,y)^2}{2(ct-L-z)} \tag{3.42}$$

$$\bar{b}(x,y,z,t) = z + \frac{(ct-z)^2 - \rho(x,y)^2}{2(ct-z)} \tag{3.43}$$

The actual integration boundaries are then chosen

$$[a,b] = \begin{cases} \emptyset, & \bar{a} < \bar{b} < 0 < D \quad (\text{Zone } I_-) \\ \emptyset, & 0 < D < \bar{a} < \bar{b} \quad (\text{Zone } I_+) \\ [0,\bar{b}], & \bar{a} < 0 < \bar{b} < D \quad (\text{Zone } II) \\ [\bar{a},\bar{b}], & 0 < \bar{a} < \bar{b} < D \quad (\text{Zone } III) \\ [\bar{a},D], & 0 < \bar{a} < D < \bar{b} \quad (\text{Zone } IV) \\ [0,D], & \bar{a} < 0 < D < \bar{b} \quad (\text{Zone } V) \end{cases} \tag{3.44}$$

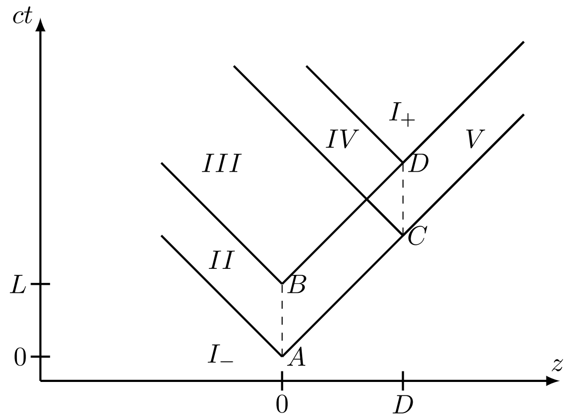


Figure 3.2: Sketch of the spacetime-diagram of a single pulse in the tz-plane. The various zones define different metric perturbations and arise from the intersection of the pulse's world sheet with the past light cone $\mathcal{J}^-$ of a spacetime event x^μ. The world sheet of the single pulse is spanned by A, B, C, D. Figure adapted from [17].

The different zones are shown in Fig. 3.2. Zones $I_\pm$ are causally disconnected from the pulse and thus the metric is not perturbed. Zone II is defined by the pulse emission from the mirror, zone III only describes the passage of the pulse (excluding emission and absorption), zone IV describes pulse absorption only, while zone V describes both emission and absorption. The world sheet of the pulse is spanned by points A, B, C, D in Fig. 3.2.

For circularly polarised pulses, we can insert the constant energy density $u(ct - z) = u_0$ into the expression for h^p to obtain

$$h^p[x, y, z, t] = \frac{4GA}{c^4} u_0 \ln\left(\frac{\zeta(x, y, z, b_{x,y,z,t})}{\zeta(x, y, z, a_{x,y,z,t})}\right) \tag{3.45}$$

where we have abbreviated e.g. $a_{x,y,z,t} := a(x, y, z, t)$. In Fig. 3.3 we plot h^p (with $\frac{4GA}{c^4} u_0$ normalised to one) for $x = y = 1$ in the tz-plane.

3.5.2 Stress-energy tensor, metric perturbation and action for two pulses

To cater for the discussion for the case of two laser pulses being present, we need to slightly extend the above considerations and specify the pulse shapes. If the effective size of the two pulses is much smaller than their separation, diffraction effects are insignificant and the total stress-energy tensor takes on the form

$$T^{\mu\nu}(t, \vec{x}) = \sum_{a=1}^{2} T_a^{\mu\nu} = A u_0 \sum_{a=1}^{2} \delta(x - x_a)\delta(y - y_a) \tag{3.46}$$

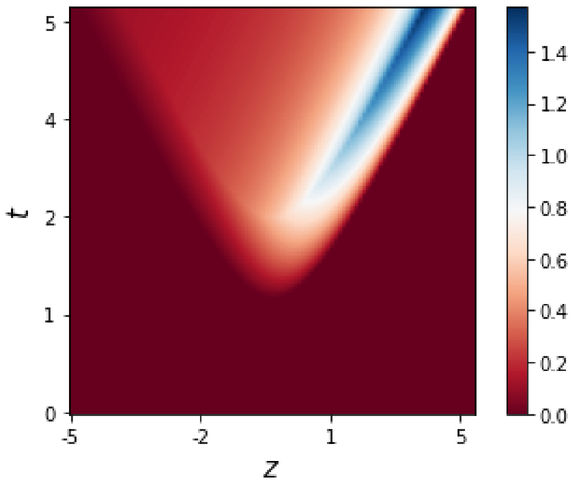


Figure 3.3: Plot of the metric perturbation $h^p[x,y,z,t]$ (with $\frac{4GA}{c^4}u_0$ normalised to one) for a single pulse at spacetime position $x = y = 1$ in the tz-plane.

where A is the effective area of each pulse in the transverse plane and (x_a, y_a) are the respective positions of the pulses in the xy-plane. For two counter-propagating light pulses propagating parallel z-coordinate axis with a sufficient separation along the x-axis with $x_2 - x_1 \neq 0$, we set $y_1 = y_2 = const$. The spacetime diagram for two counter-propagating pulses projected onto the tz-plane is plotted in Fig. 3.4.

If we assume linearised gravity, then metric perturbations due to each pulse may be simply added. In this way, interference effects have been implicitly ignored and thus dynamical effects are disregarded. This argument is sufficient to give the phase in the limit of linearised gravity. For two counter-propagating pulses along the $\hat{z}$ direction that are each located at $x_{1,2}$ along the x-axis, we obtain the total contribution to the metric by substituting the single pulse metric contributions from Eq. 3.45:

$$\begin{aligned} h^p_{tot}[x,y,z,t] &= h^p_1[x, y-y_1, z, t] + h^p_2[x, y-y_2, z, t] \\ &= h^p[x-x_1, y-y_1, z, t] + h^p[x-x_2, y-y_2, -z+D, t] \\ &= \frac{4GA}{c^4}u_0\Bigg[\ln\left(\frac{\zeta(x-x_1, y-y_1, z, b_{x-x_1,y-y_1,z,t})}{\zeta(x-x_1, y-y_1, z, a_{x-x_1,y-y_1,z,t})}\right) \\ &+ \ln\left(\frac{\zeta(x-x_2, y-y_2, -z+D, b_{x-x_2,y-y_2,-z+D,t})}{\zeta(x-x_2, y-y_2, -z+D, a_{x-x_2,y-y_2,-z+D,t})}\right)\Bigg] \end{aligned} \tag{3.47}$$

The total metric perturbation h^p_{tot} for two counter-propagating light pulses in the $(x, y = 0, z)$ plane is shown in Fig. 3.5.

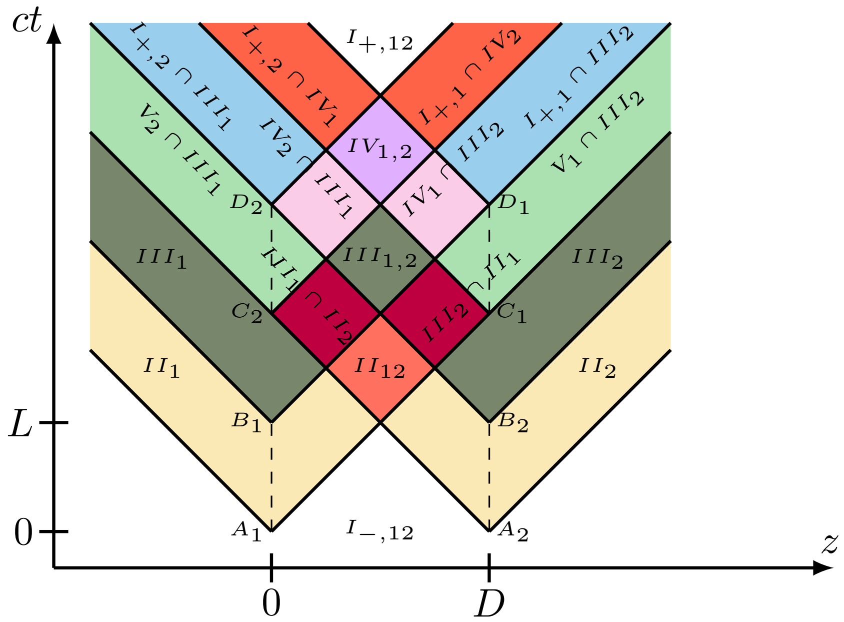


Figure 3.4: Sketch of the spacetime-diagram of two counter-propagating pulses projected onto the tz-plane. The two pulses have equal y coordinates, but different x coordinates. The various zones are found in analogy to the single pulse case.

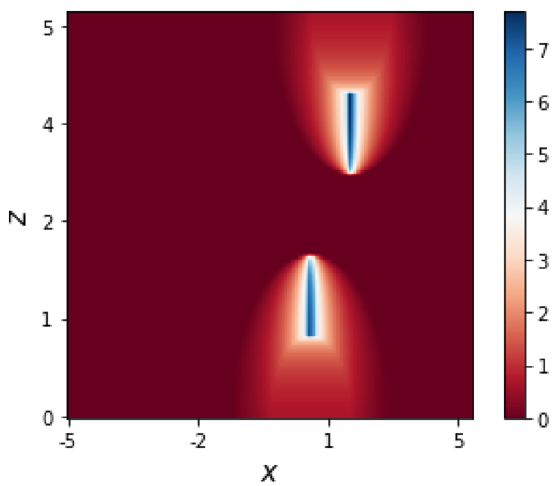


Figure 3.5: The total metric perturbation $h^p_{tot}[x, 0, z, t]$ for two counter-propagating light pulses located at $x_1 = 1, x_2 = 2$ in the $(x, y = 0, z)$ plane is shown at $t = 2$. We set $D = 5, L = 1$ (with $\frac{4GA}{c^4} u_0$ normalised to one).

3.5.3 Evaluating the action

Thus, substituting the expression for $T^{\mu\nu}$ into Eq. 3.31 for the action yields

$$
\begin{aligned}
S_h &= \frac{1}{4}\int d^4x h_{\mu\nu}T^{\mu\nu} = A\sum_{a=1}^{2}\int d^4x h^p(\vec{x},t)_{tot}u_a(z,t)\delta(x-x_a)\delta(y-y_a) \\
&= Au_0\sum_{a=1}^{2}\int dzdt h^p_{tot}(x_a,y_a,z,t) = Au_0\int dzdt\left(h^p_{tot}(x_1,y_1,z,t)+h^p_{tot}(x_2,y_2,z,t)\right) \\
&= Au_0\int_{z\in\mathbb{R}}dz\int_{t=0}^{\tau}dt\,(h^p[0,0,z,t]+h^p[x_1-x_2,0,-z+D,t]+h^p[x_2-x_1,0,z,t]+h^p[0,0,-z+D,t]) \\
&= \kappa\int_{z=-\infty}^{\infty}dz\int_{t=0}^{\tau}dt\ln\left(\frac{\zeta(0,0,z,b_{0,0,z,t})}{\zeta(0,0,z,a_{0,0,z,t})}\right)+\ln\left(\frac{\zeta(x_1-x_2,0,-z+D,b_{x_1-x_2,0,-z+D,t})}{\zeta(x_1-x_2,0,-z+D,a_{x_1-x_2,0,-z+D,t})}\right) \\
&+\ln\left(\frac{\zeta(x_2-x_1,0,z,b_{x_2-x_1,0,-z,t})}{\zeta(x_2-x_1,0,z,a_{x_2-x_1,0,z,t})}\right)+\ln\left(\frac{\zeta(0,0,-z+D,b_{0,0,-z+D,t})}{\zeta(0,0,-z+D,a_{0,0,-z+D,t})}\right)
\end{aligned}
\tag{3.48}
$$

where τ is the interaction time and $\kappa = 4GA^2u_0^2/c^4$.
While the expression in Eq. 3.48 can be evaluated numerically, we can also simplify it further analytically by considering each summand of the form $\int_{t=0}^{\tau}\int_{z=-\infty}^{\infty}dtdz\ln\left(\frac{\zeta(x,0,z,b_{x,0,z})}{\zeta(x,0,z,a_{x,0,z})}\right)$ respectively. If we do the t-integration first for fixed coordinates (x,z) we consider only the temporal evolution of $\ln\left(\frac{\zeta(x,0,z,b_{x,0,z}}{\zeta(x,0,z,a_{x,0,z}}\right)$ as in Ref. [17]. We now cover all the different zones in time t for one pulse.

- Zone I_-: $t < t_1 = \sqrt{\rho(x,0)^2+z^2}/c$ with zero contribution
- Zone II: $t_1 < t < t_2 = t_1 + \frac{L}{c}$ with non-zero contribution
- Zone III: $t_2 < t < t_3 = \frac{D}{c} + \sqrt{\rho(x,0)^2+(z-D)^2}/c$ with non-zero contribution
- Zone IV: $t_3 < t < t_4 = t_3 + \frac{L}{c}$ with non-zero contribution
- Zone V and I_+: $t > t_4$ with zero contribution

If we assume

$$
\tau > t_4 = \frac{\sqrt{\rho(x,0)^2+(z-D)^2)}+L+D}{c}
\tag{3.49}
$$

to capture all potential contributions, then we get when we consider the time-integral

$$\begin{aligned}
\int_{t=0}^{\tau} dt \ln\left(\frac{\zeta(x,0,z,b_{x,0,z,t})}{\zeta(x,0,z,a_{x,0,z,t})}\right) &= \left(\int_{II:t=t_1(x,z)}^{t_2(x,z)} + \int_{III:t=t_2(x,z)}^{t_3(x,z)} + \int_{IV:t=t_3(x,z)}^{t_4(x,z)}\right) dt \ln\left(\frac{\zeta(x,0,z,b_{x,0,z,t})}{\zeta(x,0,z,a_{x,0,z,t})}\right) \\
&= \int_{II:t=t_1(x,z)}^{t_2(x,z)} dt \ln\left(\frac{\zeta(x,0,z,\bar{b}_{x,0,z,t})}{\zeta(x,0,z,0)}\right) + \int_{III:t=t_2(x,z)}^{t_3(x,z)} dt \ln\left(\frac{\zeta(x,0,z,\bar{b}_{x,0,z,t})}{\zeta(x,0,z,\bar{a}_{x,0,z,t})}\right) \\
&+ \int_{IV:t=t_3(x,z)}^{t_4(x,z)} dt \ln\left(\frac{\zeta(x,0,z,D)}{\zeta(x,0,z,\bar{a}_{x,0,z,t})}\right) \\
&= \int_{II:t=t_1(x,z)}^{t_2(x,z)} dt \ln\left(\zeta(x,0,z,\bar{b}_{x,0,z,t})\right) - \ln\left(\zeta(x,0,z,0)\right)(t_2(x,z) - t_1(x,z)) \\
&+ \int_{III:t=t_2(x,z)}^{t_3(x,z)} dt \ln\left(1 + \frac{L}{ct-z-L}\right) + \ln\left(\zeta(x,0,z,D)\right)(t_4(x,z) - t_3(x,z)) \\
&- \int_{IV:t=t_3(x,z)}^{t_4(x,z)} dt \ln\left(\zeta(x,0,z,\bar{a}_{x,0,z,t})\right) \\
&= \int_{II:t=t_1(x,z)}^{t_2(x,z)} dt \ln\left(\zeta(x,0,z,\bar{b}_{x,0,z,t})\right) - \int_{IV:t=t_3(x,z)}^{t_4(x,z)} dt \ln\left(\zeta(x,0,z,\bar{a}_{x,0,z,t})\right) \\
&+ \int_{III:t=t_2(x,z)}^{t_3(x,z)} dt \ln\left(1 + \frac{L}{ct-z-L}\right) - \ln\left(\zeta(x,0,z,0)\right)(t_2(x,z) - t_1(x,z)) \\
&+ \ln\left(\zeta(x,0,z,D)\right)(t_4(x,z) - t_3(x,z)) \\
&= \int_{II:t=t_1(x,z)}^{t_2(x,z)} dt \ln(ct-z) - \int_{IV:t=t_3(x,z)}^{t_4(x,z)} dt \ln(ct-z-L) \\
&+ \int_{III:t=t_2(x,z)}^{t_3(x,z)} dt \ln\left(1 + \frac{L}{ct-z-L}\right) - \ln\left(\zeta(x,0,z,0)\right)\frac{L}{c} + \ln\left(\zeta(x,0,z,D)\right)\frac{L}{c} \\
&= \left(-t + (t - \frac{z}{c})\log(ct-z)\right)\Big|_{t_1(x,z)}^{t_2(x,z)} - \left(-t + (t - \frac{L+z}{c})\log(ct-z-L)\right)\Big|_{t_3(x,z)}^{t_4(x,z)} \\
&+ \frac{1}{c}\left((ct-z)\log\left(\frac{-ct+z}{L-ct+z}\right) + L\log(L-ct+z)\right)\Big|_{t_2(x,z)}^{t_3(x,z)} - \ln\left(\zeta(x,0,z,0)\right)\frac{L}{c} \\
&+ \ln\left(\zeta(x,0,z,D)\right)\frac{L}{c}
\end{aligned} \tag{3.50}$$

as $\zeta(\bar{a}) = ct - z - L$ and $\zeta(\bar{b}) = ct - z$ and we have taken out the terms that have no t-dependence as constants. Here we have also used

$$\int dt \ln(ct-z) = -t + (t - \frac{z}{c})\log(ct-z) \tag{3.51}$$

$$\int dt \ln(ct-z-L) = -t + \frac{ct-L-z}{c}\log(ct-z-L) \tag{3.52}$$

$$\int dt \ln(1+\frac{L}{ct-z-L}) = \frac{1}{c}\Big((ct-z)\log(\frac{-ct+z}{L-ct+z}) + L\log(L-ct+z)\Big) \tag{3.53}$$

Finally, the z integration can be done analytically to yield the respective contributions to the action in Eq. 3.48. As defined in Ref. [109], the phase to be attached to each polarisation configuration σ (and corresponding spatial path) is given by

$$\phi_\sigma = \frac{S^\sigma_{\mathcal{F}}[x_a, \mathcal{F}[x_a]]}{\hbar} \tag{3.54}$$

We have the following values of $x_{1,2}$ for the configurations corresponding to the different spatial branches:

	x_1	x_2
LL	0	d
LR	0	$d+\Delta x$
RL	Δx	d
RR	Δx	$d+\Delta x$

Table 3.1

The spatial state will evolve to

$$\begin{aligned} |\psi(\tau)\rangle &= \frac{1}{2}\Big(e^{i\phi_{LL}}|LL\rangle + e^{i\phi_{LR}}|LR\rangle + e^{i\phi_{RL}}|RL\rangle + e^{i\phi_{RR}}|RR\rangle\Big) \\ \propto \frac{1}{2}\Big(|LL\rangle &+ e^{i(\phi_{LR}-\phi_{LL})}|LR\rangle + e^{i(\phi_{RL}-\phi_{LL})}|RL\rangle + e^{i(\phi_{RR}-\phi_{LL})}|RR\rangle\Big) = \\ &\frac{1}{2}\Big(|LL\rangle + e^{i(\phi_{LR}-\phi_{LL})}|LR\rangle + e^{i(\phi_{RL}-\phi_{LL})}|RL\rangle + |RR\rangle\Big) \end{aligned} \tag{3.55}$$

where we have split off a global phase and used $\phi_{RR} = \phi_{LL}$, as only relative phases determine the degree of entanglement.
Consider the relative phase factor $e^{i(\phi_{RL}-\phi_{LL})}$ for $|RL\rangle$ at closest approach with

$$\begin{aligned} \hbar(\phi_{RL}-\phi_{LL}) = Au_0 \int_{z=-\infty}^{\infty} dz \int_{t=0}^{\tau} dt \\ (h^p[d-\Delta x, 0, -z+D, t] + h^p[d-\Delta x, 0, z, t] - h^p[d, 0, -z+D, t] - h^p[d, 0, z, t]) \end{aligned} \tag{3.56}$$

using $h^p[x,\cdot] = h^p[-x,\cdot]$. Analogously,

$$\begin{aligned} \hbar(\phi_{LR}-\phi_{LL}) = Au_0 \int_{z=-\infty}^{\infty} dz \int_{t=0}^{\tau} dt \\ (h^p[d+\Delta x, 0, -z+D, t] + h^p[d+\Delta x, 0, z, t] - h^p[d, 0, -z+D, t] - h^p[d, 0, z, t]) \end{aligned} \tag{3.57}$$

3.5.4 Numerically calculating the phase evolution

To numerically integrate $h^p[x, y, z, t]$, the necessary z-integration limits increase with the interaction time τ. As is shown in Fig. 3.6 by the integrand of the relative phase, $\phi_{RL} - \phi_{LL}$ is computed by taking those z values into account that are still causally connected to the emission of the two light pulses up to interaction time τ.

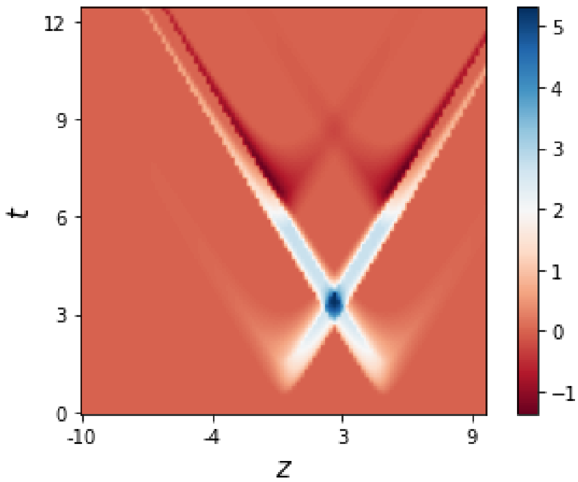


Figure 3.6: Plot of the integrand of the relative phase $\phi_{RL} - \phi_{LL}$ (with $\frac{4GA}{c^4}u_0$ normalised to one) for two counter-propagating pulses with $D = 5$ and $L = 1$ in the tz-plane.

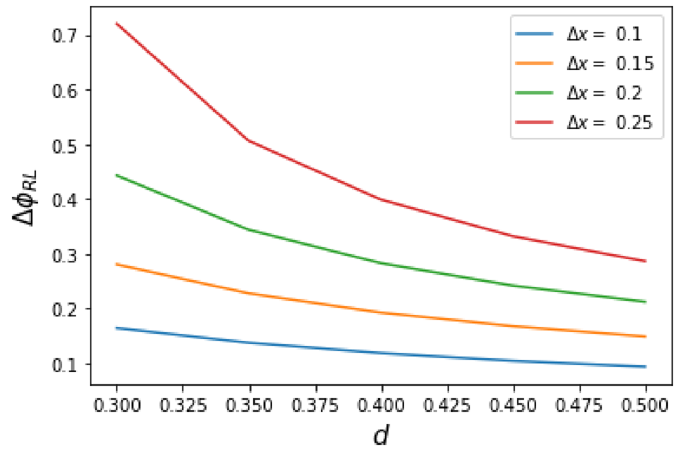
As one can extract from Fig. 3.6, the integrand of the relative phase converges eventually as the interaction time τ is increased. This is, however, only a finite size effect due to the length of the interferometer D limiting the gravitational interaction. By increasing the length of the interferometers D, the phase evolution calculation would incorporate larger sections of the full spacetime integral $\int_{z=-\infty}^{\infty}\int_{t=0}^{\tau} dt dz$.

To fully appreciate the high amount of phase induction between the two light pulses, one would need to increase the MZ interferometer length D. As this requires integration over a large spacetime volume, the numerical brute-force computation becomes quite challenging quickly. Unfortunately, no sophisticated methods have been devised in this work to find estimates for the phases being induced for arbitrarily large interaction times $\tau = \frac{D}{c}$. To simplify the calculations further, we will only consider in the following the spacetime volume confined to the duration of the pulse[9].

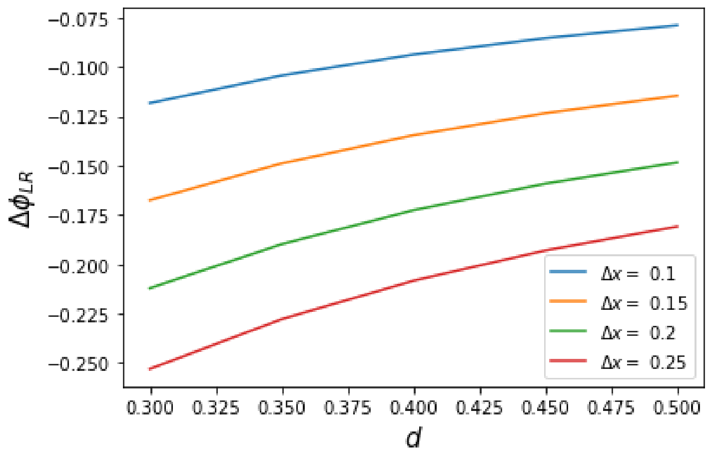
[9]There is also a small contribution to the phase for larger times.

3.5.5 Parameter dependence of phase

First, we numerically investigate the parameter dependence of $\Delta\phi_{RL}$ and $\Delta\phi_{LR}$ on the length of the interferometers D, the splitting distance Δx and the separation between the interferometers d. In Fig. 3.7 we show the dependence for various $(\Delta x, d)$. As can be seen, the phase induction is maximal if $d - \Delta x$ is small.



((a))



((b))

Figure 3.7: Plot of the relative phase for two counter-propagating light beams at closest approach (a)$\Delta\phi_{RL}$ and for (b)$\Delta\phi_{LR}$ at fixed interferometer length D examining the dependence on the splitting distance Δx and interferometer separation d. We have normalised $\frac{4GA}{c^4}u_0$ to one. The parameters were chosen as: $D = 1, L = 0.1, \tau = 2$.

3.6 Phase estimation

First, consider the constant $\kappa = 4GA^2u_0^2/c^4$ in Eq. 3.48. Since each phase scales with the corresponding action divided by $\hbar$ we have $4G/(c^4\hbar) = 3.14\cdot 10^{-10}$ (units omitted). To simplify calculations, let us assume the closest phase approximation

$$d - \Delta x \ll d, \Delta x \tag{3.58}$$

such that only the RL term contributes. It is to be noted that for the calculation to be valid, diffraction effects must be excluded, i.e. we must operate in a regime where $d - \Delta x \gg \lambda$ where λ is the wavelength. Then, assuming we have chosen $d, D, L, \Delta x$ such that we have a phase $\Delta\phi_{RL}$ of order 1, we require $u_0^2A^2 \approx 10^{10}$. Certainly, this is only a sketch of the possible parameters we would use in such an experiment and a more detailed analysis may be the subject for future research.

3.6.1 Stability of spatial branches and effect of gravitational deflection

During the interaction, the gravitational interaction will not only leave a mark on the relative phase induced but will also contribute to the gravitational deflection of the pulses' trajectories in space. If the gravitational deflection is non-negligible, then we would need to integrate over the change of position along the coordinate axis aligned with the axis of separation of two respective pulses. Henceforth, we would like to investigate if this effect needs to be considered or not.

While the aim is to show the effect of one laser pulse on a counter-propagating laser pulse, it is sufficient to consider the effect on a single test particle freely falling in the laser pulse gravitational field, where we follow the approach in Ref. [17]. The line element of spacetime is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu \tag{3.59}$$

and the worldline $\gamma^\mu(\lambda)$ of the test particle parametrised by λ is governed by the geodesic equation:

$$\ddot{\gamma}^\mu + \Gamma^\mu_{\rho\sigma}\dot{\gamma}^\rho\dot{\gamma}^\sigma = 0 \tag{3.60}$$

where $\dot{\gamma} = \frac{d\gamma}{dt}$ for null geodesics (i.e. $\lambda = t$) and $\Gamma^\mu_{\rho\sigma}$ is the Christoffel connection

$$\Gamma^\mu_{\rho\sigma} = \frac{1}{2}g^{\mu\nu}(g_{\sigma\nu,\rho} + g_{\nu\rho,\sigma} - g_{\rho\sigma,\nu}). \tag{3.61}$$

For massless particles we have $g_{\mu\nu}\dot{\gamma}^\mu\dot{\gamma}^\nu = 0$. Now substituting $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ into the expression for the line element yields

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \\ &= (\eta_{00} + h_{00})dx^0dx^0 + \eta_{xx}dx^x dx^x + \eta_{yy}dx^y dx^y + (\eta_{zz} + h_{zz})dx^z dx^z + (h_{0z} + h_{z0})dx^0 dx^z \\ &= (\eta_{00} + h^p)dx^0dx^0 + \eta_{xx}dx^x dx^x + \eta_{yy}dx^y dx^y + (\eta_{zz} + h^p)dx^z dx^z - 2h^p dx^0 dx^z \\ &= -(1-h^p)c^2dt^2 + (1+h^p)dz^2 - 2h^p c dt dz + dx^2 + dy^2 \end{aligned} \tag{3.62}$$

Converting this into light-cone coordinates $u := ct - z$ and $v := ct + z$ we obtain:

$$ds^2 = -du dv + h^p du^2 + dx^2 + dy^2 \tag{3.63}$$

with geodesic equations

$$\begin{aligned} \ddot{\gamma}^z &= \frac{1}{2}\partial_z h^p(\dot{\gamma}^u)^2 + \frac{d}{d\lambda}(h^p\dot{\gamma}^u) \\ \ddot{\gamma}^x &= \frac{1}{2}\partial_x h^p(\dot{\gamma}^u)^2 \\ \ddot{\gamma}^y &= \frac{1}{2}\partial_y h^p(\dot{\gamma}^u)^2 \end{aligned} \tag{3.64}$$

and subsidiary condition for null geodesics:

$$-\dot{\gamma}^u\dot{\gamma}^v + h^p(\dot{\gamma}^u)^2 + (\dot{\gamma}^x)^2 + (\dot{\gamma}^y)^2 = 0 \tag{3.65}$$

where $\gamma^u = \gamma^0 - \gamma^z, \gamma^v = \gamma^0 + \gamma^z$. Here we have taken h^p to be simply the metric perturbation due to one pulse, as is sufficient for an order-of-magnitude derivation.
As stated in Ref. [17], there is a rotational symmetry of the metric perturbation around the z-axis, but no symmetry in the azimuthal direction, which means we can focus on the xz-plane. Eventually, the geodesic equation for acceleration along the x-axis is

$$\ddot{\gamma}^x = \frac{1}{2}\partial_x h^p(\dot{\gamma}^0 - \dot{\gamma}^z)^2 \tag{3.66}$$

For a massless test particle moving in the negative z direction, we have $\dot{\gamma}^0 = c = -\dot{\gamma}^z$. By inserting this into the geodesic equation for the acceleration along the x-axis yields:

$$\ddot{\gamma}^x = 2c^2\partial_x h^p \tag{3.67}$$

which amounts to a non-zero gravitational force acting on the test particle. However, as shown in Ref. [17], the magnitude of acceleration amounts to $10^{-18} m/s^2$ for current conventional lasers. Hence, we will not consider gravitational deflection any further, as it is a negligible effect.

Chapter 4

Conclusions and outlook

4.1 Summary

The search for experimental hints on the path to a unified theory of quantum gravity has recently advanced substantially by a proposal aimed to witness the gravity mediated entanglement between two massive objects [14, 15]. Even though there is still considerable debate regarding the actual conclusions one can draw from the experiment on the non-classicality of gravity in the research community, a substantial amount of work has been done to improve on the original proposal.

While the gravitational coupling of two highly intense counter-propagating laser beams sent through Mach-Zehnder interferometers is very small, this work has examined their potential use for the protocol to testify gravity mediated entanglement. We considered the basics of the experimental implementation using two Mach-Zehnder interferometers and derived the metric perturbation sourced by the two light pulses inspired by Ref. [17] using linearised gravity. By adopting the considerations in Ref. [109] to derive the phases using path integrals, we found the action of the pulses and studied the effect on the gravitational phase being induced.

In addition, our analysis showed that there is a priori no in-principle reason why our proposal cannot serve as the first scheme to be realised to show the gravitational coupling of light beams through entanglement generation instead of spatial deflection. Nevertheless, a detailed investigation into the feasibility needs to be done as further research. As laid out below, there are several considerations that should be further investigated.

4.2 Further considerations

While we have treated the two pulses as two monochromatic, spatially separated counter-propagating light pulses, one may also consider having two overlapping light pulses for the protocol. If the two pulses come close to each other, we can no longer treat them as being specified by their effective area only. While in some papers light pulses are only assumed to be 'thin' pencils [16] or cylindrical beams of finite radius [113], both models lack diffraction. However, when two light pulses are close to each other, then diffraction is inevitable as the light beams interact. This corresponds to the short-wavelength limit, where all wavelike properties of light are neglected. While [17] took into account finite wavelengths, diffraction was not considered. To fill this gap, Schneiter et al [85] investigated the metric sourced by such a Gaussian beam using a perturbative expansion in the beam divergence as a solution to Maxwell's equation. This may be a promising pathway for future research as it opens up the possibility to consider modelling the photonic GME protocol at even smaller beam separations and arbitrary (more realistic) beam shapes. Since higher order terms in the beam divergence were shown to source the gravitational deflection of two co-propagating pulses, the photonic GME protocol may also be run with two co-propagating pulses increasing the entanglement generation further[1].

Moreover, the assumption of monochromatic light beams is quite unrealistic and one should extend the calculations by considering a bandwidth of frequencies. In addition, decoherence is probably the most limiting factor of the experiment given the long interaction times. Each photon loss or absorption in the beam within a fibre contributes to destroying the superposition state[2]. Dephasing might arise from phase noise in the laser or thermal fluctuations.
Similarly, the light pulses may also be embedded into a (waveguide) medium for which the speed of propagation of light is smaller than in vacuum, which means that also the co-propagating setup would induce a phase [79]. The entanglement growth would then possibly be enhanced due to longer interaction times. On the other hand, the laser power in a medium may also be limited and decoherence may be a stronger limiting factor. In analogy to the generalisation to arbitrary geometries and qudits for the massive GME protocol [25, 26], one may also consider an array of light beams to boost the gravitational phase induction. In this manner, it may be interesting to examine whether there is any setup that entangles faster than the one proposed in this dissertation and investigate how many measurement runs are required to establish a statistically significant value for the entanglement witness[3].

To increase the gravitational phase induction, we can also generalise the Mach-Zehnder interferometer setup by winding up the fibres in loops during the relevant stage for GME. Along a circle, we emit two light pulses by an angle of π apart, so the light

[1]One difficulty that may arise is the question of whether the derivation in Section 3.4 for the unitary time evolution operator is still valid as the light pulses need to be in two distinguishable spatial modes.

[2]This might be overcome by instead of increasing the interaction time, exploiting the repeatability of the experiment. One can increase the number of measurements and use statistical analysis [18] to witness a small accumulated phase due to the gravitational interaction.

[3]This may be used to rule out non-local gravity theories.

beams are counterpropagating both in z and y direction always. Since y is now no longer fixed, we need to include this in the calculation. We can generalise this to l loops by simply adding the actions additively to multiply the phase by a factor $2l$ for l loops, where D is the length of half a loop. While this calculation ignored the complications that arise with this geometry, it shows the possibility of evolving enough phase due to more contrived geometries. Alternatively, we may take into account a more accurate derivation of the metric perturbations for the MZ setup including loops. During the protocol (as it may be very long), we should check the motional deflection of the beams and make it negligible. We should also check the decoherence time of the cat state.

Not only does the photonic implementation open up a potentially new window into investigating gravity mediated entanglement with increased accuracy, but using photons as opposed to massive objects in a near-future experiment could also shed light on other theoretical aspects/features of gravity that cannot be witnessed using the latter.

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